Applied Mathematics

Dynamics

3rd Secondary

Concepts Sheet

Differentiation and Integration of vector functions:

- If each of s , v, x are functions in the time then $v = \frac{dx}{dt} = \frac{ds}{dt} \Rightarrow s = \int vdt \ , \ x = \int vdt$
- $a = \frac{dv}{dt} \Rightarrow v = \int adt$
- If a is a function in the position, then $a = v\frac{dv}{dx} \Rightarrow \int adx = \int vdv$
- If a is a function in the displacement, then $a = v\frac{dv}{ds} \Rightarrow \int ads = \int vdv$
- The average velocity: $v_a = \frac{Total\ distance}{Total\ time}$, The average velocity vector $= \frac{Total\ displacement}{Total\ Time}$
- The motion is accelerated if ($va > 0$)
- The motion is (decelerated) if ($va < 0$)

The momentum of a body at a moment is a vector quantity whose magnitude is equal to the product of the mass of this body by its velocity at this moment and its direction is the direction of the velocity itself

$\therefore \vec{H} = m\vec{v}$

The change of the momentum of a body $= m(\vec{v_2} - \vec{v_1})$

$\Delta H = m\int_{t_1}^{t_2} a\ dt$

If the acceleration a is a function of time t

Newton's First Law

Every body preserves in its state of rest or of moving uniformly unless acted upon by an unbalanced external force by an external effect

Newton second law

A body whose mass is m and moves with a uniform acceleration a

$ma = \mathbf{F}$ where **F** is the resultant of the forces acting on the body.

- If $\boldsymbol{a} = \frac{\boldsymbol{dv}}{\boldsymbol{dt}}$, then the equation of motion is in the form:

  $\int_{t_1}^{t_2} F\,dt = m\int_{v_1}^{v_2} dv$

- If $\boldsymbol{a} = \boldsymbol{v}\frac{\boldsymbol{dv}}{\boldsymbol{ds}}$, then the equation of motion is in the form:

  $\int_{s_1}^{s_2} F\,ds = m\int_{v_1}^{v_2} v\,dv$

- If the mass is variable, then the equation of motion is in the form: $\mathrm{F} = \frac{\boldsymbol{d}}{\boldsymbol{dt}}(m\,v)$

The units used with the equation of motion

m(kg). $\boldsymbol{a}$ (m/sec^2) = F (newton)

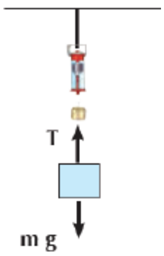
m(gm) $\boldsymbol{a}$ (cm/sec^2) = F (dyne)

Applications on Newton's laws of moving a body inside a lift

- The lift at rest or moving with a uniform velocity $\boldsymbol{mg} = \boldsymbol{N}$
- The lift ascends with acceleration $(\boldsymbol{a})$: $\boldsymbol{N} - \boldsymbol{mg} = \boldsymbol{ma}$
- The lift descends with acceleration $(\boldsymbol{a})$: $\boldsymbol{mg} - \boldsymbol{N} = \boldsymbol{ma}$

  Where N (appearing weight), (reading of the balance) or (reaction of the lift's ground)

Applications on Newton's laws of a body suspended in a spring balance



- The lift at rest or moving with a uniform velocity $\boldsymbol{mg} = \boldsymbol{T}$
- The lift ascends with acceleration $(\boldsymbol{a})$: $\boldsymbol{T} - \boldsymbol{mg} = \boldsymbol{ma}$
- The lift descends with acceleration $(\boldsymbol{a})$: $\boldsymbol{mg} - \boldsymbol{T} = \boldsymbol{ma}$

  Where T (is the tension in the spring balance)

Applications on Newton's laws of the lift

- The lift at rest or moving with a uniform velocity $m'g = T$
- The lift ascends with acceleration (a): $T - m'g = m'a$
- The lift descends with acceleration (a): $m'g - T = m'a$

Where T (is the tension in the rope which carrying the lift)

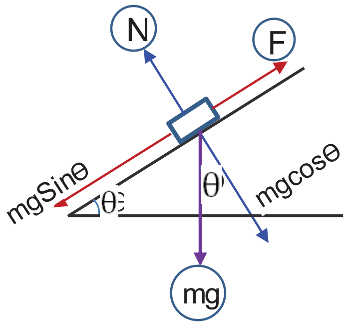
, m' is the total mass (lift + inside)

Remarks:

- If the appearing weight > real weight, then the lift is moving upwards with a uniform acceleration or moving down with a uniform deceleration.
- If the appearing weight < real weight, then the lift is moving down with a uniform acceleration or moving Up with a uniform deceleration

Motion of a body of mass (m) moving on a smooth inclined plane

inclines by an angle of measure $\theta°$ to the horizontal



- If $F > mg \sin \theta$

  Then body moves with a uniform acceleration (a) upwards the plane.

  and the equation of its motion is $F - mg \sin\theta = ma$
- If $F < mg \sin\theta$

  then body moves with a uniform acceleration (a) downwards the plane and the equation of its motion is $mg \sin\theta - F = ma$

Motion of a body of mass (m) on a rough inclined plane

rises by an angle of measure $\theta°$ to the horizontal,

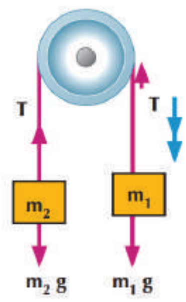
μ_K is the kinetic friction coefficient.

- If the motion is Upwards:

  Then, the equation of its motion is : $F - mg \sin \theta - \mu_K mg \cos \theta = ma$
- If the motion is downwards:

  Then, the equation of its motion is : $mg \sin \theta - F - \mu_K mg \cos \theta = ma$

Simple pulley

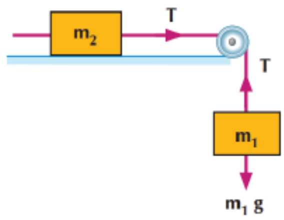


- Equations of motion

  $m_1 a = m_1 g - T$

  $m_2 a = T - m_2 g$

  The pressure on the pulley = 2 T

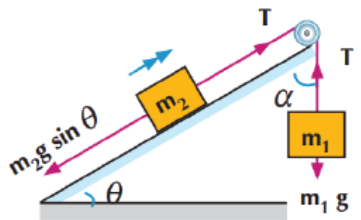


- Equations of motion

  $m_1 a = m_1 g - T$

  $m_2 a = T$

  The pressure on the pulley = $\sqrt{2}$ T



- Equations of motion

  $m_1 a = m_1 g - T$

  $m_2 a = T - m_2 g \sin\theta$

The pressure on the pulley = 2 T $cos\frac{\alpha}{2}$

$$=T\sqrt{2 + 2sin\,\theta}$$

Impulse and momentum

If a constant force $\vec{F}$ acts on a body an interval of time t ∈ $[t_1, t_2]$ then the impulse of this force I = F × t

If a variable force F (function in time) acts on a body during interval of time

t ∈ $[t_1, t_2]$,then the impulse I = $\int_{t_1}^{t_2} Fdt = m(v_2 - v_1)$ = change in momentum

Elastic collision: If the deformation does not occur, heat is not generated and there is not loss in the kinetic energy. $m_1 \overrightarrow{v`_1} + m_2\overrightarrow{v`_2} = m_1 \overrightarrow{v_1} + m_2\overrightarrow{v_2}$
i.e: the sum of the two momentums after the collision directly = the sum of the two momentums before the collision directly. If two smooth balls collide, then the sum of their two momentums does not change due to the collision. Algebraic measures can be used as follows: $m_1\, v`_1 - m_1\, v_1 = -I, m_2\, v`_2 - m_2\, v_2 = I$
$m_1\, v`_1 + m_2\, v`_2 = m_1\, v_1 + m_2\, v_2$ Since I is the algebraic measure to the impulse of the second ball on the first ball v_1, $v_2$are the algebraic measure of the velocities before collision $v`_1$ , $v`_2$are the velocities after collision.

The direct collision: The two velocities before and after the collision directly are parallel to the line of the two centers at the moment of collision.

Inelastic collision: The inelastic collision is meant that a deformation takes place, heat is generated, or the bodies get contacted due to the collision process (loss of kinetic energy occurs). In spite of it all, the momentum after and before collision remains as it is without change. and the momentum conservation equation is form: (in case the two bodies contacted):

$m_1\, \overrightarrow{v_1} + m_2\overrightarrow{v_2} = (m_1 + m_2)\vec{v}`$ (by using vectors in case of soldering)

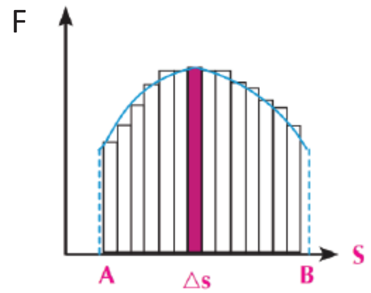
$m_1\, v_1 + m_2\, v_2 = (m_1 + m_2)v`$ (by using algebraic measures)

Work done by a fixed force (F) to move a body from initial position to a terminal position w $= \vec{F} \cdot \vec{S} = \|\vec{F}\|\|\vec{S}\|cos\theta$

Where θ is the measure of the smallest angle between $\vec{F}$, $\vec{S}$ and the force is constant , then:

- $\vec{F}$, $\vec{S}$ have the same direction then $W = FS$
- $\vec{F} \perp \vec{S}$ then $W = Zero$
- $\vec{F}$, $\vec{S}$ have two opposite directions $W = -FS$

The work done by a variable force(F) parallel to the direction of the motion where F is a function in the displacement to move a body from S=A to the point S=B $W = \int_A^B F dS$



Units of measuring Work:

Joule (Newton .m) $=\mathbf{10^7}$erg (dyne.cm) , Kg. wt. m=9.8 joule

Kinetic energy:

The kinetic energy of the body is the energy which the body acquires due to its velocity, and it is estimated at a moment as the product of half the mass of the particle times the square of the magnitude of its velocity and it is denoted by the symbol T. If m is the mass of a particle, $\vec{v}$ is its velocity vector and v is the algebraic measure of this vector, then:

$$T = \frac{1}{2} m ||\vec{v}||^2 = \frac{1}{2} mv^2$$

$$T = \frac{1}{2} m (\vec{v} . \vec{v})$$

Units of kinetic energy is the same as work units

The principle of work and energy

The change of the kinetic energy of a particle as it transfers from an initial position to a final position is equal to the work done by the force acting on it during the displacement between those two positions.

$T - T_o = W$

The potential energy : $P = mg S$

The change of the potential energy of a particle is equal to negative the work done

$P_B - P_A = -W$

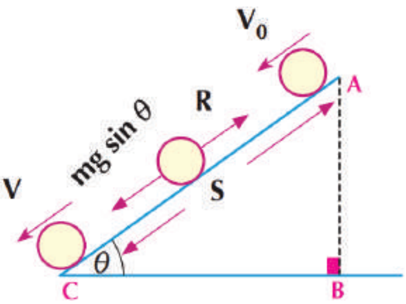
Conservation of energy

Conservation of energy

If a body moves from position A to another position B without encountering any resistance, then the sum of the kinetic and potential energies at A is equal to the sum of the kinetic and potential energies at B. From the principle of work and energy, we find that: $T_B + P_B = T_A + P_A$

sum of the kinetic and potential energies remains constant during the motion

Motion on a rough inclined plane



If a body descends on a rough inclined plane under the action of its weight only from position A to position C, then the change in the potential energy = the change in kinetic energy + the work done against resistances.

Power (P):

is the rate of change of the work with respect to the time if the force is constant

$$P = \frac{dW}{dt} = \vec{F}.\vec{V}$$

: Horse =75 Kg. wt m/sec = 75 × 9.8 Newton .m/sec (watt)

Work done = $\int_{t_1}^{t_2} Power\ dt$